

February 2008

# Multifluid Models for Cyclic Cosmology

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## Abstract

Inspired by the Landau two-fluid model of superfluidity, we consider a similar multifluid description for cosmology where two normal fluids occur for matter and radiation respectively. For cyclic cosmology, two dark energy superfluid components turn out to be insufficient but three superfluids can lead to a sensible five-fluid model which in a certain limit becomes indistinguishable from the brane-world cyclic model proposed earlier (Baum and Frampton). Distinguishing more general five-fluid models from brane-world models for cyclic cosmology could be feasible with more accurate observational data.

*Introduction.* In the history of physics, it is impossible to exaggerate the fecundity of cross-fertilization between sub-disciplines. In theoretical physics, high-energy physics and cosmology have been repeatedly informed by condensed matter theory. One outstanding example is the idea of spontaneous symmetry breaking introduced by Nambu [1] into particle theory inspired by study of the BCS theory [2] of superconductivity. The origin of the idea of spontaneous symmetry breaking is in the Bogoliubov microscopic theory of superfluidity [3], where the nonvanishing expectation value of the field describes the superfluid component of the Bose liquid; this idea was exploited also in the theory of superconductivity [4]. Few ideas have had more impact on our understanding of both high energy physics and cosmology.

Here we take our inspiration for study of cyclic cosmology [5–7] from the Landau two-fluid model [8] of superfluidity, itself also applicable to superconductivity [4, 9]; the microscopic theory of superfluidity was developed by Bogoliubov [3]. In this model, the energy density of superfluid liquid helium is expressed as a sum of two terms

$$\rho = \rho_n + \rho_s, \quad (1)$$

where  $\rho_n$  is for the normal component and  $\rho_s$  is for the superfluid component.

At the lambda temperature, only normal fluid is present. As temperature is decreased, more and more normal fluid is converted to superfluid until at absolute zero the liquid helium consists only of superfluid. Normal fluids behave like a Newtonian fluid with viscosity and entropy. The superfluid components have no viscosity, no entropy and do not carry any heat. The normal fluid and superfluid satisfy different equations of motion.

Similarly the different cosmological fluids will satisfy different equations of state. One dark energy superfluid (density  $\rho_1$ , equation of state  $w_1$ ) is the one currently measurable. Of the others,  $\rho_2$  ( $w_2$ ) becomes important very close to the turnaround and  $\rho_3$  ( $w_3$ ) is important very close to the bounce.

Thus, in this analogy it is natural to associate superfluids with dark energy because it has no entropy. The normal fluids will correspond with the matter and radiation. Thus there are  $n_n = 2$  normal fluids for cosmology, and we shall show that, for cyclic cosmology, we need  $n_s = 3$  dark energy superfluids, making overall a five-fluid model. Another approach to a dark energy fluid is in [10].

*Friedman equation.* We write the Friedman equation as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[ \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} + \sum_{i=1}^{i=n_s} (\rho_i)_0 a(t)^{3\phi_i} \right], \quad (2)$$

where the equations of state for the normal fluids are  $w_m = 0$  and  $w_r = +1/3$  for matter and radiation respectively. The number of superfluids representing dark energy is  $n_s$  and the first component with “ $i = 1$ ” will be the presently observed dark energy with  $w_1 = -1 - \phi_1$  and  $\phi_1 > 0$ .

In Eq.(2), the  $n_s$  terms in the summation are analogs of the superfluid term in the Landau theory in that they carry zero entropy. To implement cyclic cosmology, it will be necessary that some (actually those with  $i \geq 2$ ) superfluid energy densities ( $\rho_i$ ) be negative.

Let us first consider an  $n_s = 2$  four-fluid model with  $\phi_2 > \phi_1$ , that is  $w_2 = -1 - \phi_2 < w_1$ . For turnaround from expansion to contraction at time  $t = t_T$ , and using  $a(t_0) = 1$ , we see that

$$(\rho_2)_0 = -(\rho_1)_0(a(t_T))^{-3(\phi_2 - \phi_1)}, \quad (3)$$

so that  $(\rho_2)_0 < 0$  and, because  $a(t_T) \gg 1$  and if  $(\phi_2 - \phi_1)$  is sufficiently non-zero, it follows that  $|(\rho_2)_0| \ll |(\rho_1)_0|$  and hence that the second (“ $i = 2$ ” in Eq.(2)) dark energy superfluid is unobservably small at the present time  $t = t_0$ .

Can  $\rho_2$  play the role of causing both the turnaround and the bounce in cyclic cosmology? The answer is negative as is now explained by a no-go theorem.

*No-Go theorem.* Let us prove a no-go theorem that  $n_s = 2$  cannot produce an acceptable bounce at  $t = t_B$  where contraction turns into expansion.

At the bounce when  $t = t_B$ , we would need

$$(\rho_2)_0 = -(\rho_r)_0(a(t_B))^{-(4+3\phi_2)} \quad (4)$$

and, because  $a(t_B) \ll 1$ , this would require  $\phi_2 < -4/3$ , or  $w_2 > +1/3$ , clearly inconsistent with Eq.(3) which requires  $\phi_2 > 0$  and  $w_2 > +1/3$ . This incompatibility of Eqs.(3) and (4) provide a No-Go theorem for any four-fluid ( $n_n = 2$  and  $n_s = 2$ ) model.

This is not surprising when we consider the brane-world Friedman equation [5]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[ \rho_\Lambda a(t)^{3\phi_\Lambda} + \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} - \frac{\rho_{total}^2}{\rho_C} \right], \quad (5)$$

where  $\rho_{total} = (\rho_\Lambda + \rho_m + \rho_r)$ . Thus, the final term on the right-hand-side of Eq.(5) has quite a different time dependence for  $t \rightarrow t_T$  and  $t \rightarrow t_B$ . This underlies the No-Go theorem and mandates usage of the following five-fluid model.

*Five-Fluid Model.* We consider a five-fluid model with  $n_n = 2$  and  $n_s = 3$ . Redefine the densities  $\rho_2 \rightarrow -\rho_2$  and  $\rho_3 \rightarrow -\rho_3$  and the Friedman equation becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[ \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} + (\rho_1)_0 a(t)^{3\phi_1} - (\rho_2)_0 a(t)^{3\phi_2} - (\rho_3)_0 a(t)^{3\phi_3} \right], \quad (6)$$

and in this case we can arrange that at the turnaround

$$(\rho_2)_0 = (\rho_1)_0 (a(t_T))^{-3(\phi_2 - \phi_1)}, \quad (7)$$

and at the bounce

$$(\rho_3)_0 = (\rho_r)_0 (a(t_B))^{-(4+3\phi_3)}. \quad (8)$$

At the turnaround, as in the four-fluid model, we require (I)  $(\phi_2 - \phi_1) > 0$  while at the bounce there is the new condition (II)  $(\phi_3 + 4/3) < 0$ .

Taking these two inequalities (I) and (II) together will ensure the turnaround and bounce occur and that  $|(\rho_1)_0| > |(\rho_2)_0|$  and  $|(\rho_r)_0| > |(\rho_3)_0|$ , as necessary to preserve the successful description of the present universe. For special values of the equations of state  $w_2$  and  $w_3$  the five-fluid model becomes indistinguishable from the BF model, as follows.

*Brane-world as special case of five-fluid model.* In the special case, consistent with the above inequalities, where  $\phi_2 = 2\phi_1$  and  $\phi_3 = -8/3$  the Friedman equation of Eq.(6) become indistinguishable for that of the brane-world model in Eq.(5). Although Eq.(6) is not *identical* to Eq.(5) there is no observable difference because at present the components  $(\rho_2)$  and  $(\rho_3)$  are completely negligible; at turnaround  $(\rho_2)$  duplicates the final term in Eq.(5) and at the bounce  $(\rho_3)$  plays precisely the same role.

However, the five-fluid model is more general if we incorporate arbitrary values of  $\phi_2$  and  $\phi_3$  consistent with the above inequalities (I) and (II).

*Distinguishing models.* Let us consider a five-fluid model which is very disparate from the BF model. In such a case, accurate observations can distinguish the cyclic models.

Of course, we do not yet know  $\phi_1$  precisely for the dark energy but let us suppose that  $\phi_1 = 0.05$ , consistent with present WMAP data [11]. We need  $\phi_2$  to be bigger so let us assume  $\phi_2 = 0.06$ . In this case Eq.(7) requires that

$$(\rho_2)_0 = (\rho_1)_0 a(t_T)^{0.03}. \quad (9)$$

Just to complete an example, let us now assume  $a(t_T) = 10^{33.33}$  whereupon Eq.(9) dictates that  $(\rho_1)_0 = 10(\rho_2)_0$  and so the fit to dark energy should use a scale dependence

$$(\rho_{DE})_0 [a(t)^{0.15} - 0.1a(t)^{0.18}] . \quad (10)$$

More generally the multifluid model suggests fitting to

$$(\rho_{DE})_0 [a(t)^{3\phi_1} - \eta a(t)^{3\phi_2}] , \quad (11)$$

where  $\phi_2 > \phi_1$  and  $\eta$  is an additional parameter related, in general, to the turnaround scale.

More accurate and complete data on dark energy will enable distinction between fitting with a two-term formula like Eq.(11) and fitting with only the first term.

*Discussion* Inspired by previous successes, we have here attempted to emulate the two-fluid model of superfluidity in a five-fluid model of cyclic cosmology. The analogy is heightened by the zero entropy for the dark energy (superfluid) components,

The multifluid models have certain advantages, including that they do not necessitate derivations from brane worlds in higher dimensionality. The analogies to the two-fluid model of superfluidity may be posited directly in four dimensions.

In a certain limit, the five fluid model with two normal fluids, matter and radiation, and three superfluids for dark energy become indistinguishable from the model of ref. [5].

However, when the five-fluid model become very disparate from the brane-world model it will be possible to distinguish them by accurate observations of dark energy as we have discussed.

It is therefore worth examining, as more and better observational data become available, whether fits to a two-term expression as in Eq.(11) are more successful for dark energy than those using only the first term thereof.

In conclusion, cosmological dark energy is perhaps the most important theory challenge in physics or astronomy and has stymied all attempts at its understanding. Although history does not always repeat itself, it does

seem well worth studying the research works in condensed matter theory by *e.g.* John Bardeen, Bogoliubov and Landau. This is precisely what provoked our present suggestion that dark energy be best regarded as a superfluid, or superposition of superfluids, of the type very familiar in condensed matter. This, in turn, hints at an alternative to the big bang initial singularity.

## Acknowledgements

The work of PHF and SM was supported in part by the U.S. Department of Energy under Grant No. DG-FG02-06ER41418. IA was supported in part by the INTAS grant 03-51-6346.

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